HOMEWORK 3

DUE MONDAY, FEBRUARY 1, 2010

Sakurai Problems: 23, 25, 27, 28

Spin $\frac{1}{2}$ Particle in a $\vec{B}(t)$ field

Consider a single spin-1/2 particle prepare in the $+\hbar/2$ eigenstate of S_z.

a) What are the amplitudes $c_\uparrow(t), c_\downarrow(t)$ of finding the spin in the $+\hbar/2, -\hbar/2$ states respectively after a weak perturbation due to a time-dependent magnetic field pulse in the following cases:
Case I. $\vec{B}(t) = \hat{z}B_o\tau\delta(t)$
Case II. $\vec{B}(t) = \hat{y}B_o\tau\delta(t)$

Note: The interaction term is $V(t) = -\mu_B\vec{\sigma}\cdot\vec{B}(t)$

b)Repeat the above analysis for a finite length pulse i.e.
$|\vec{B}(t)| = 0$ *when* $t < 0$ *and when* $t > \tau$
$|\vec{B}(t)| = B_o$ *when* $0 < t < \tau$

(You should get the part a) result in the limit of an infinitely short time i.e. the delta-pulse)

c) There is some pulse duration τ^* at which this 1^{st} order perturbation theory breaks down. Find it from examining your results.

d) Solve this problem exactly (much like in class!) for $c_\uparrow(t), c_\downarrow(t)$ and compare the lowest order Taylor expansion with your perturbation solutions from above. For Case II ($\vec{B}||\hat{y}$) calculate the magnetic field pulse duration τ_1 for which the spin is in the $\downarrow$ state with certainty. Compare this to τ^* from part c). What does this mean?